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Is the distribution of pessimistic states of mind a market failure?[i]

Takanobu IKEDA※

Abstract

The market mechanism, or the price adjustment mechanism, and the state of mind have been considered important factors in economic activities, but how these two are related has not been explored. The former is called the Walrasian mechanism and is the basic framework of microeconomics, in which psychological factors are put behind the scenes or considered restrictions to rational decision-making by agents. The latter is argued as a main factor in a trade cycle in macroeconomics. Because microeconomics and macroeconomics have not been integrated into a fully synthesized theory, the distribution of states of mind has not been regarded as a factor in price adjustment in a market economy or as a market failure. We consider what kind of state of mind an agent can have based on past market trading and which factors bind his trading opportunity. This paper presents a model of Samuelsonian neoclassical synthesis, in which either the Walrasian mechanism or a non-Walrasian quantity-constrained mechanism works depending on the distribution of states of mind. We claim that in many cases, a non-Walrasian mechanism is at work and with some policies, the Walrasian mechanism would be restored.

Key Words: microfoundations of macroeconomics, quantity-constrained behavior, neoclassical synthesis, market failure, resource allocation mechanism.

1. Introduction

The market mechanism, or the price adjustment mechanism, and the state of mind have been considered important factors in economic activities, but how these two are related has not yet been explored. The former is called the Walrasian mechanism and is the basic framework of microeconomics, in which psychological factors are put behind the scenes

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※Professor in the Department of Political Sciences and Economics at Takushoku University.
e-mail: tikeda@ner.takushoku-u.ac.jp

of, and considered restrictions to rational decision-making by agents. The latter is argued as a main factor in a trade cycle in macroeconomics. Because microeconomics and macroeconomics have not been integrated into a fully synthesized theory,[1] the distribution of states of mind has not been regarded as a factor in price adjustment in a market economy or as a market failure. This paper presents a model of Samuelsonian neoclassical synthesis, in which either the Walrasian mechanism or a non-Walrasian quantity constrained mechanism works depending on the distribution of states of mind. We claim that in many cases, a non-Walrasian mechanism is at work and with some policies, the Walrasian mechanism would be restored.

The concept of the state of mind comes in a market economy model as follows. In the Walrasian mechanism, an agent is small enough to be an atomic existence and is a price-taker.[2] He believes that he has a limitless trading opportunity, perceiving that he can trade as much as he wants at the prevailing prices. He maximizes his utility and sends quantity signals of demand and supply. These quantity signals are used only in the process of price adjustment. It is known as the existence theorem in which under certain conditions of classical economic environments, a perfect economic system, the market structure called by perfect competition, the Walrasian mechanism yields a market equilibrium. And the fundamental theorem of welfare economics claims that a Walrasian equilibrium allocation is Pareto optimal. In other words, in a situation which lacks of any condition for the existence theorem and the fundamental theorem of welfare economics, a market equilibrium allocation need not be optimal. There is a market failure and some policies are necessary to mitigate the market failure. We diverge from this story by one point: that is, an agent in this paper receives not only price signals but also quantity signals sent by the other agents, and perceives his trading opportunity based on signals.[3] His past experiences in market trading form how he perceives his trading opportunity. We refer to a type of perception as a state of mind. We examine how the distribution of states of mind affects the price adjustment mechanism in a market economy, and we claim that the distribution of states of mind could be a market failure. In other words, the model in this paper provides a microfoundation of macroeconomics.

Today, there is widespread belief that a framework of dynamic general equilibrium analy-

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1 Samuelson called neoclassical synthesis the integration of microeconomics and macroeconomics. It is known that he used the word "neoclassical synthesis" for the first time in the third version of his textbook *Economics: An Introductory Analysis* in 1955.

2 Leijonhufvud (1968), pp. 68-69, characterizes a perfect competitive market as one with the following two conditions. The first condition is that all traders are too small to have capability of manipulating the prices. The second condition is that the demand and supply curves are perfectly elastic. In this paper, we call the first condition the requirement of "atomic existence" and the second condition the requirement of "limitless trading opportunity." Leijonhufvud calls a market an "atomistic market" where the first condition is satisfied but the speed of price adjustment is not instantaneous.

3 The pioneering works of quantity constrained equilibrium models by Clower (1965) and Leijonhufvud (1968) present general equilibrium models in which quantity signals sent by traders play some role in traders' trading opportunities. See Hahn (1978) and Gale (1978) for models in which a trader conjectures his trading opportunity based on quantity signals sent by the other traders.

sis should provide enough microfoundation of macroeconomics.[4] This approach is based on the belief that the price adjustment mechanism works at full-strength, but that some market failures invalidate the price adjustment mechanism and the economy as a whole falls into "pathological" conditions. However, at a time when many young economists became enthusiastic over Keynesian economics as an infectious disease spreads, it was strongly believed that the price adjustment mechanism had some problems; hence prices are fixed or inflexible, and the Keynesian world shows itself.[5] Keynesian economics of the quantity adjustment mechanism replaced classical economics of the price adjustment mechanism. Following the post-war prosperity of the American economy, P. A. Samuelson coined the term "neoclassical synthesis," which "combines the essentials of the theory of aggregative income determination with the older classical theories of relative prices and of microeconomics." In a healthy economy, with monetary and fiscal policies operating to validate the high-employment assumption postulated by the classical theory, that classical theory comes back into its own.[6] As Samuelson himself admits, "many qualifications to this optimistic formulation"[7] are needed. Because different mechanisms are at work in these two worlds, they cannot be combined. In this paper, we generalize the Walrasian equilibrium model into an integrated one, which closes "the cleavage between microeconomics and macroeconomics."[8]

In the Walrasian mechanism, an agent is a price-taker. He receives price signal, and does not perceive any limit on his trading. Only the auctioneer receives quantity signals concerning demand and supply sent by all the agents and adjusts the prices. In this paper, an agent receives quantity signals as well as price signals, and perceives his trading opportunity based on signals. How much he perceives to trade depends on his past experiences with market trading. If he had successes in the past, he supposes an optimistic trading opportunity. If everyone is optimistic, the world is governed by a Walrasian price adjustment. If he has failed in the past, he may suppose a pessimistic trading opportunity,[9] If all traders are pessimistic, the Keynesian world appears, in which prices are fixed and only

4 The microfoundation of macroeconomics is an attempt to draw macroeconomic implications from the microeconomic description of agents' behavior. Early attempts consist of two approaches. One is the aforementioned quantity constrained equilibrium approach. The other is an approach of imperfect information by Alchian (1970), Lucas=Lapping (1970), and others in Phelps (1970). The former was followed by Barro-Grossman (1971), Benassy (1975), Drèze (1975), and Younès (1975) of quantity constrained models with fixed or inflexible price assumption. This developed into the neo-Keynesian theory of monopolistic competition models. The latter developed into models of the rational expectation hypothesis. In later years, the framework of dynamic general equilibrium presented by Sargent (1979, 1993) was at the center of discussions on microfoundation. It is postulated that the price adjustment mechanism operate most efficiently.

5 See, for example, Harris (1948).

6 See Samuelson (1955), p. 733.

7 See Samuelson (1955), footnote 1, p. 360.

8 See Samuelson (1955) p.360.

9 In this paper, psychological factors are confined to affect agents' trading opportunities. Thus, experiences from past events are the main determinants of a state of mind. A trader's state of mind concerning future events strongly affects his preference.

quantities are adjusted.[10] In most cases, it is a mixture of Walrasian and Keynesian worlds. In this paper, we claim that the distribution of pessimistic states of mind affects the effectiveness of the price adjustment mechanism.

We will consider a temporary equilibrium framework in which there is no adjustment information over future events, because we would not be pessimistic if we had full information on future environments.[11] From past experiences, we draw a picture of the present and future states of the world. Therefore, past experiences are a part of our environments. On the basis of past experiences in market trading and quantity signals sent by the other agents, an agent perceives his trading opportunity. In this paper, we use the remaining demand, which is the sum of the quantity signals sent by the other traders with a minus sign, to describe a trader's state of mind. If an agent perceives that he can trade more than the remaining demand, we say that he is optimistic. If an agent perceives that he can trade as much as the remaining demand, we say that he is pessimistic. If an agent perceives that he can trade less than the remaining demand, we say that he is extremely pessimistic.[12] The distribution of states of mind or traders' perceptions of their trading opportunities determines the degree of price flexibility. There are extremes: one is the Walrasian world and the other is the Keynesian world.

This paper is organized as follows. In Section 2, we summarize the development of quantity-constrained models. In Section 3, we present a market economy model in which an agent is under a state of mind concerning his trading opportunity. In Section 4, we prove the existence of a market equilibrium under that state of mind. In Section 5, we discuss the properties of market equilibria. When all traders are optimistic a market is under a Walrasian mechanism which yields a Walrasian equilibrium. However, under the other distribution of states of mind the price adjustment mechanism deteriorates according to the degree of mixture of states of mind. That is, most distributions of states of mind yield non-Walrasian equilibria at which traders are quantity-constrained. This is the market failure. In Section 6, we argue the necessity of three policy steps to remedy this market failure.

2. A short note on quantity-constrained equilibrium models

In this section, we describe the development of quantity-constrained equilibrium models briefly. Clower (1965) presents a quantity-constrained equilibrium model and claims that it provides the microfoundations of Keynesian economics. In a monetary economy, if a trader is bounded by a quantity constraint in one market, particularly the labor market,

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10 As for macroeconomic implications of quantity constrained models, see Barro-Grossman (1971), Malinvaud (1977), and Benassy (1982).

11 A temporary equilibrium framework proposed by Hicks (1939). See Grandmont (1982).

12 Ikeda (1984) shows that in a game-theory framework, even with the existence of an auctioneer the price adjustment mechanism does not work and results in quantity- constrained equilibria if it is assumed that the feasibility condition holds and a trader trades as much as his quantity signal. In other words, only pessimistic state of mind in this paper is possible in such a framework and no Walrasian world appears. As for quantity- constrained models in a game-theory framework, see Böhm-Lévine (1979) and Heller-Starr (1979).

this changes his trade decisions of the other goods. This is the spillover effect. Leijonhufvud (1968) searches out the microfoundations of Keynesian economics in several aspects. Among them, he emphasizes the absence of an auctioneer in a real market, based on Arrow (1959), and claims that trade quantities adjust much faster than prices. In that case, a trader is naturally quantity-constrained.

In response to Clower (1965) and Leijonhufvud (1968), quantity-constrained equilibrium models are studied in Europe under the assumption that the prices are fixed or move in limited bounds. Depending on how to interpret the Clower mechanism, Benassy (1975), Drèze (1975), and Younès (1975) show different kinds of a general equilibrium model. Grandmont (1982) uses a temporary equilibrium framework so that traders have different expectations about their future and are effectively under quantity constraints. Using these quantity-constrained equilibrium models in the macroeconomic context, Barro-Grossman (1971) and Malinvaud (1976) attempt to relate these models with Keynesian economics. Presenting aggregate models, they construct a Keynesian world in which monetary and fiscal policies have a place to work.

Negishi (1974, 1979) points out that a trader's trades may be restricted by subjective trading possibilities, and not by objective quantity limits. He claims that if traders' subjective demand curves are kinked, then their trades are under quantity constraints and prices are hardly flexible. Hahn (1978) proposes a non-Walrasian equilibrium model in which traders have conjectures about trade limits and end up being quantity-constrained. The point Hahn (1978) wants to make is that Walrasian equilibria and non-Walrasian equilibria exit in his model. Using Hahn's non-Walrasian equilibrium model, Gale (1978) claims that if all conjecture functions are differentiable, then any equilibrium, if it exists, is Walrasian. In this sense, kinks also work in Hahn's non-Walrasian equilibrium model.

The studies described above introduce quantity constraints into the Arrow-Debreu (1954) type of a general equilibrium model. In contrast, Böhm-Lévine (1979) and Heller-Starr (1979) consider trade games. In game theory, all outcomes, whether in equilibrium or not, must be feasible. In a trade game, regardless of what the proposed trades are, the sum of realized trade quantities over all traders must be zero. We call this the feasibility condition or the balance condition. Under this condition, traders are naturally quantity-constrained.

Ikeda (1984) examines Hahn's non-Walrasian equilibrium approach in the framework of a game theory. By considering an outcome function, which corresponds proposed trade quantities to a realized one as a conjecture function a la Hahn, he attempts to locate the intrinsic properties of a conjecture function. The first requirement on a conjecture function is the feasibility condition, that is, the balance condition. The second requirement is that any trade proposal must be credible. That is, if the sum of all proposed trade quantities over all traders is zero, - that is, proposed gross excess demands is zero,- then a trader's realized trade coincides with his proposed one. That is, he has to buy as much as he ordered, or sell as much as he promised when all traders' proposed quantities satisfy the balance condition. From these two conditions, Ikeda (1984) shows that a conjecture function may be own-differentiable but not cross-differentiable. That is, the intrinsic kinkedness in a quantity-constrained equilibrium model is cross-indifferentiable. After Clower (1965), many quantity-constrained equilibrium models reasonably classify trades into various domains, such as recession, deflation, inflation, and reflation, and argue that a market outcome may be different depending on the domain trade.

Ikeda (1984, 1986) shows that, at any market price a variety of quantity-constrained equilibria exist. Even at a Walrasian equilibrium price, not only Walrasian equilibrium trades but also many non-Walrasian equilibrium trades exist. Because such non-Walrasian equilibria, that are quantity-constrained equilibria provide microfoundations of Keynesian economics, the coexistence of Walrasian and non-Walrasian equilibria may be regarded as a representation of neoclassical synthesis by Samuelson (1948). In this paper, we say it is the distribution of states of mind that leads to which types of equilibrium, Keynesian or Walrasian.

3. Model

We consider an exchange economy $E = (X^i, u^i, \omega^i)$ in the framework of a temporary equilibrium. There are n consumers ($N = \{1,\cdots,n\}$) and ℓ goods ($L = \{1,\cdots,\ell\}$), where the ℓ-th good is money. The consumption set is a subset of the ℓ dimensional Euclidean space. The preference ordering can be representable by a utility function $u^i : X^i \to R$. We assume the following assumptions on these.

Assumption (1) Agent i's consumption set X^i is close, convex, and bounded from below.
Assumption (2) Agent i's utility function $u^i : X^i \to R$ is continuous, strongly quasi-concave, and has no bliss point.
Assumption (3) Agent i's initial endowment $\omega^i \in \mathrm{int}\, X^i$.

In the following, we consider a net trade $z^i = x^i - \omega^i \in Z^i$, $x^i \in X^i$. Given a price vector $p \in \Delta^\ell = \{p \in R^\ell \mid p \geq 0, \sum_h p_h = 1\}$, agent i has to satisfy the budget constraint. That is,

$$p \cdot z^i = 0.$$

In the Walrasian mechanism, agents determine their demands, and hence trades, depending on only a price signal. However, because every agent sends a trade message z^i, he must receive some trade quantity signals. That is, agent i receives a signal $s^i = (p, z(i)) \in \Delta^\ell \times \prod_{k \neq i} Z^k$, where z(i) stands for $z(i) = (z^1,\cdots,z^{i-1}, z^{i+1},\cdots,z^n)$. Receiving quantity signals from other agents as well as the price signal, an agent conjectures his trading possibility and maximizes his utility subject to his conjecture.

An agent perceives his trading possibility in the following way. First, he has a "trading possibility target" for every good except for the ℓ-th good, money.

Definition 3.1 For each good $h = 1,\cdots, \ell-1$, agent i has a trading possibility target which is the value of the following trade quantity function $F^i : \prod_{k \neq I} Z^k \to R^{\ell-1}$. That is,

$$F_h^i\ (z_h(i)) = -\lambda_h^i \textstyle\sum_{k \neq i} z_h^k, \qquad 0 \leqq \lambda_h^i.$$

$-\sum_{k \neq i} z_h^k$ represents agent i's remaining demand, because $\sum_{k \neq i} z_h^k$ is the sum of all agents' trading proposals except for his own proposal. $0 \leqq \lambda_h^i$ represents agent i's state of mind, which shows his prospect of realizable trading intensity. The agent has his λ^{i} based on his experiences of past trades. Depending on the value of λ^{i}, agent i's state of mind can be classified in the following way.

Definition 3.2 Agent i's "state of mind" for good h is

(i) "extremely pessimistic," if $0 \leqq \lambda_h{}^{i} < 1$.

(ii) "pessimistic," if $\lambda_h{}^{i} = 1$.

(iii) "optimistic," if $1 < \lambda_h{}^{i} < \infty$, and

(iv) "extremely optimistic," if $\lambda_h{}^{i} = \infty$.

It is significant that the state of mind is defined for each good, because an agent may have different experiences of trades for different goods. An agent may have succeeded in trading a good beyond his expectation. At the same time, he may have failed in trading another good unexpectedly.

In case (i), agent i perceives that his trading possibility is less than the remaining demand. In case (ii), his trading possibility is considered the same as the remaining demand. In case (iii), he perceives that his trading possibility is more than the remaining demand and he believes that he can realize it. In the Walrasian mechanism, there are only extremely optimistic agents. There is neither an extremely pessimistic agent, nor a pessimistic agent.

In the Walrasian mechanism, every agent is assumed to be successful in trading; hence he had successful experiences in the past. Depending on past successes, he is eager and ambitious to be successful in trading at present as well as in the future. Consequently, he can be extremely optimistic. However, in a real world, a number of people have had past failures, and cannot be optimistic, especially if the failure is fatal, in which case one may lose one's motive to trade and may become extremely pessimistic.

The trading possibility target is the remaining demand multiplied by parameter $\lambda_h{}^{i}$, that is, agent i's state of mind. An agent can trade a quantity between the upper and lower bounds, one of which is the trading possibility target and the other bound is zero.

Definition 3.3 Agent i's "trading possibility" of good h consists of the upper and lower bounds. That is,

$$z_h{}^{i}{}_{-} = \min\,[0, F_h{}^{i}\ (z_h)] \leqq 0 \ \ \&\ \ z_h{}^{i-} = \max\,[0, F_h{}^{i}\ (z_h)] \geqq 0.$$

Now agent i perceives his trading possibility by the upper and lower bounds of his trade quantity. This perception may have the following consequences. First, if he is extremely pessimistic, he keeps sending his trading proposal at less than his Walrasian excess demand, so that he will exit from the market at a market equilibrium. If he is pessimistic, his trading quantity signals are always the same as the remaining demand, so that the market reaches equilibrium at an instance where the demand equals the supply. In these two cases, Walrasian excess demands of the agents in question have nothing to do with price adjustment. Only if the agent is optimistic, his quantity signal is greater than or equal to the remaining demand, so that it coincides with the Walrasian demand at a market equilibrium. This is how the agent's state of mind affects his quantity signal, according to which the price is adjusted. Sometimes the distribution of the agents' states of mind disturbs the price adjustment.

An agent determines his quantity signal to maximize his utility subject to his trading possibility constraint as well as his budget constraint. Now, we describe these constraints. Agent i receives the signal $s^i = (p, z(i)) \in S^i = \Delta^{\ell} \times R^{(n-1)(\ell-1)}$, and perceives his trading possibility set as follows.

Definition 3.4 Agent i's "trading opportunity correspondence" $\beta^i : S^i \rightarrow Z^i$ consists of his budget constraint and his trade possibility. That is,

$$\beta^i(s^i) = \{z^i \in Z^i | \ p \cdot z^i = 0, z_h{}^i{}_- \leqq z_h{}^i \leqq z_h{}^{i-}, h \neq \ell\}.$$

This trading opportunity correspondence has the following nature.

Proposition 1

Suppose that a consumption set satisfies assumption (1) and is bounded. If the initial endowment satisfies assumption (3), then the trading opportunity correspondence is continuous at a signal s^i and is convex-valued.

Proof

Note that the trading opportunity correspondence is the product of two correspondences. That is,

$$\beta^i(s^i) = \beta^{Wi}(s^i) \cap C^i(s^i),$$

where $\beta^{Wi}(s^i) = \{z^i \in Z^i | \ p \cdot z^i = 0\}$ and $C^i(s^i) = \{z^i \in Z^i | \ z_h{}^i{}_- \leqq z_h{}^i \leqq z_h{}^{i-}, h \neq \ell\}$. $\beta^{Wi}(s^i)$ is the ordinary Walrasian budget constraint set. Because of Assumption (3), at any price vector $p \in \Delta^{\ell}$, there exists $x^i \in X^i$ such that $p \cdot x^i < p \cdot \omega^i$ or $p \cdot z^i < 0$. Thus, by the well-known theorem, $\beta^{Wi}(s^i)$ is continuous at p. It is obvious that $\beta^{Wi}(s^i)$ is continuous at s^i and convex-valued.

The correspondence $C^i(s^i)$ is such that a continuous function $F^i(z_h(i))$ of $z(i) \in R^{(n-1)(\ell-1)}$ is either the upper or lower bound and 0 is the other bound, that is the graph of $C^i(s^i)$ is a rectangular parallelepiped. Thus, $C^i(s^i)$ is continuous at s^i and is convex-valued.

Because both $\beta^{Wi}(s^i)$ and $C^i(s^i)$ are continuous at s^i and are convex-valued, so is $\beta^i(s^i)$.

Q.E.D

Agent i chooses his excess demand z^i so as to maximize his utility subject to $\beta^i(s^i)$. In the following, we use a utility index $v^i : Z^i \rightarrow R$ defined by $v^i(z^i) = u^i(x^i + \omega^i)$. By assumption (2), a utility index $v^i(z^i)$ is continuous and strongly quasi-concave.

Definition 3.5 Given a signal s^i, the "constrained excess demand correspondence" ζ^i : $S^i \rightarrow Z^i$ is defined by $\zeta^i(s^i) = \{z^i \in \beta^i(s^i) \ | \ z^i \text{ maximizes } v^i(z^i)\}$.

Because a utility function is strongly concave, the excess demand correspondence $\zeta^i(s)$ is a function. Now, the nature of an excess demand correspondence is as follows:

Proposition 2

Suppose that a consumption set satisfies assumption (1) and is bounded. If a utility function satisfies assumption (2) and the initial endowment satisfies assumption (3), then the excess demand function ζ^i (s^i) is non-empty and continuous at a signal s^i.

Proof

Because a consumption set is closed and bounded by assumption (1), it is compact. Hence, by Proposition 1 the trading opportunity correspondence β^i (s^i) is continuous at s^i and is convex-valued. Thus, the set β^i (s^i) is non-empty and compact. Because a utility function is continuous by assumption (2), there exists a maximum by the well-known Weierstrass theorem. Thus, the constrained excess demand correspondence ζ^i (s^i) is non-empty and continuous at s^i.

Q.E.D.

4. Existence of a market equilibrium under the state of mind λ[13]

We need to see whether there exists a market equilibrium under the state of mind λ when agents behave in the way described in Section 3. If a market works in the same way as the Walrasian mechanism, we define a market equilibrium as follows:

Definition 4.1

A "market equilibrium under the state of mind λ" is $(p^*, (\mathbf{z}^{i*}))$ which satisfies the following two conditions:

(i) for every $i = 1,\cdots,n$, $\mathbf{z}^{i*} = \zeta^i$ (s^{i*}),

(ii) $z^* = \sum_i z_h{}^{i*} \leqq 0$, $p^* \cdot z^* = 0$.

Formally these two conditions are exactly those that define a Walrasian equilibrium. However, an agent determines his quantity signal depending not only on price signals but also on his state of mind and other agents' quantity signals in this mechanism. This is the only difference from the Walrasian mechanism, Therefore, the existence theorem of a market equilibrium under the state of mind λ is very similar to that of a Walrasian equilibrium.

Existence Theorem

Assume the following assumptions for all i:

Assumption (1′) The set of excess demands Z^i is closed, convex, and bounded from below.

Assumption (2′) A utility function v^i:$Z^i \rightarrow R$ is continuous, strongly concave, and has no bliss point.

Assumption (3′) $0 \in \mathrm{int} Z^i$

Then, there exists a market equilibrium (p^*, z^*) under the state of mind λ.

Proof

13 See Arrow-Debreu (1954), Debreu (1959), Arrow-Hahn (1971). Also see Fukuoka (1979) for the proof in this section.

As we use the fixed-point theorem, it is required for Z^i to be compact. Because the set is bounded from below by assumption (1'), the set of excess demands that satisfy the feasibility condition $z = \sum_r x^r - \sum_r \omega^r \leqq 0$ is obviously bounded. Now, we define the set K by $K = \{k \in R^{\ell} \,\|k^r| \leqq c \text{ for all } r\}$, where c is sufficiently large. By applying the fixed-point theorem for the set $\tilde{Z}^i = Z^i \cap K$, we prove that there exists a market equilibrium. Then we show that the market equilibrium with the sets $(\tilde{Z}^i)$ continues to be a market equilibrium with the sets (Z^i).

First, we note that the trade opportunity correspondence and the excess demand function have the characterisitics shown in Propositions 1 and 2 on the subset $\tilde{Z}^i$.

We also note that the excess demand function for the whole economy $\tilde{\zeta}(s) = \sum_r \tilde{\zeta}^r(s^r)$ is continuous for the set $\tilde{Z}^i$.

Next, because all the agents satisfy budget constraints $p \cdot z^i = 0$, Walras Law

$$p \cdot \sum_r z^r = p \cdot z = 0$$

holds for all $z = \tilde{\zeta}(s)$.

Whereas the excess demand function of the whole economy represents the quantity adjustment, a price adjustment function represents the price adjustment by the auctioneer.

Following the law of demand and supply, the auctioneer raises the price if the excess demand is positive, and reduces the price if the excess demand is negative.

We use a price adjustment function that describes such price adjustment process. That is,

$$\eta_h(p, z) = \frac{p_h + \max(z_h, 0)}{\sum_{t=1}^{\ell}[p_t + \max(z_t, 0)]} \qquad h = 1, \cdots, \ell.$$

Then, $\eta = (\eta_1, \ldots, \eta_\ell) : \varDelta \times \tilde{Z} \rightarrow \varDelta$ is a continuous function.

By coupling the price adjustment function with the excess demand function, the mapping $\eta(p, z) \times \tilde{\zeta}(s) : \varDelta \times \tilde{Z} \rightarrow \varDelta \times \tilde{Z}$ is a continuous function from a non-empty, compact, convex set to a non-empty, compact, convex set. Thus, by applying Brouwer's fixed-point theorem, there exists a fixed point (p^*, z^*) such that $p^* = \eta(p^*, z^*)$ and $z^* = \tilde{\zeta}(s^*) = \tilde{\zeta}(p^*, z^*)$.

As Walras Law $p^* \cdot z^* = 0$ holds, it remains to show that $z^* \leqq 0$ in order to show that condition (ii) of a market equilibrium holds. However, as described below, this follows from $p^* = \eta(p^*, z^*)$.

$$p_h^* \sum_{t=1}^{\ell} [p_t^* + \max(z_t^*, 0)] = p_h^*[1 + \sum_{t=1}^{\ell} \max(z_t^*, 0)] = p_h^* + \max(z_h^*, 0)$$

$$p_h^* \sum_{t=1}^{\ell} \max(z_t^*, 0) = \max(z_h^*, 0)$$

Multiplying both sides by z_h^* and adding over h, we obtain

$$\sum_{h=1}^{\ell} p_h^* z_h^* \sum_{t=1}^{\ell} \max(z_t^*, 0) = \sum_{h=1}^{\ell} z_h^* \max(z_h^*, 0).$$

By applying Walras Law on the left side,

$$0 = \sum_{h=1}^{\ell} z_h^* \max(z_h^*, 0) = \sum_{h=1}^{\ell} z_h^* \max(z_h^*, 0) = \sum_{h=1}^{\ell} \max((z_h^*)^2, 0)$$

The last equality holds because of the nature of max function. Thus, we obtain

$$\sum_{h=1}^{\ell} \max(z_h^*, 0) = 0, \text{ or } z_h^* \leqq 0, \qquad h=1.\cdots, \ell.$$

Now condition (ii) is satisfied.

It remains to show that $z^* = \tilde{\zeta}(s^*) = \tilde{\zeta}(p^*, z^*)$, that is, to show that $z^{i*} = \tilde{\zeta}^i(s^{i*})$ maximizes agent i's utility subject to his constraint for all $i = 1.\cdots, n$. Now, suppose that z^{i*} does not satisfy condition (i) of a market equilibrium, that is, $z^{i*} \neq \zeta(s^{i*})$. Then there must exist $\mathbf{z}^{i\prime}$ such that $\mathbf{z}^{i\prime} \in \beta^i(s^{i*}) = \{z^i \in Z^i | p^* \cdot z^i = 0, z_h{}^i{}_-{}^* \leqq z_h{}^i \leqq z_h{}^{i-*}, h \neq \ell\}$ and $v^i(z^{i*}) < v^i(\mathbf{z}^{i\prime})$. By defining $z^i(\alpha)$ by $z^i(\alpha) = \alpha\mathbf{z}^{i\prime} + (1-\alpha)z^{i*}, 0 < \alpha < 1$, we have $z^i(\alpha) \in \beta^i(s^{i*})$ because $\beta^i(s^{i*})$ is a convex set. By the quasi-concavity of a utility function, $v^i(z^{i*}) < v^i(z^i(\alpha))$. By taking α sufficiently close to 0, we have α such that $z^i(\alpha) \in \tilde{Z}^i$. This contradicts $z^{i*} = \tilde{\zeta}^i(s^{i*})$. Hence $z^{i*} = \zeta^i(s^{i*})$ holds. This is condition (i) of a market equilibrium.

The above shows that the fixed point (p^*, z^*) satisfies conditions (i) and (ii) of a market equilibrium. Now (p^*, z^*) is a market equilibrium under the state of mind λ.

Q.E.D.

In the following sections, we explore the nature of a market equilibrium under the state of mind λ. To do so, we heavily use the characteristic of a market equilibrium shown in Proposition 3. To state Proposition 3, we need the following definition.

Definition 4.2

The market equilibrium $(p^*, (\mathbf{z}^{i*}))$ under the state of mind λ with $(\mathbf{z}^{i*}) = (0)$, that is $(p^*, (0))$ is called a "trivial market equilibrium."

Proposition 3

(1) If there exists a market equilibrium $(p^*, (\mathbf{z}^{i*}))$ under the state of mind λ, for all i and all h, then $z_h{}^{i*} = -\sum_{k \neq i} z_h{}^{k*}$.

(2) For any λ and any p, a trivial market equilibrium $(p, (z^i)) = (p, (0))$ is a market equilibrium under the state of mind λ.

Proof

By condition (ii) of a market equilibrium, for all h, $\mathbf{z}_h^* = \sum_i \mathbf{z}_h^{i*} \leqq 0$. To show statement (1) by contradiction, we suppose that there exists some h with $\mathbf{z}_h^{i*} < -\sum_{k \neq I} \mathbf{z}_h^{k*}$. Then, there exists j such that $\mathbf{z}_h^{j*} < 0$. Because of the supposition $\mathbf{z}_h^* = \sum_i \mathbf{z}_h^{i*} < 0$ and Walras Law $p^* z_h^* = 0, p^* = 0$. Then agent j's utility increases by replacing $\mathbf{z}_h^{j*}$ by $\mathbf{z}_h^{j\prime}$ with $\mathbf{z}_h^{j\prime} > \mathbf{z}_h^{j*}$. This contradicts equilibrium condition (ii).

Now, we show statement (2). With a trivial allocation $(z^i) = (0)$, $z_h{}^i{}_- = 0 = z_h{}^{i-}$ for all h and i. Then, for all i agent i's trading possibility set consists of only one element, that is, no trade. $\beta^i(p, (0)(i)) = (0)$. For all i, agent i's constrained excess demand correspondence consists of no trade, that is, $\zeta^i(p, (0)(i)) = (0)$. Thus, $(p, (0))$ satisfies equilibrium condition (i). Because $p \cdot 0 = p \cdot \sum_i 0 = 0$, $(p, (0))$ satisfies equilibrium condition (ii).

Q.E.D.

Statement (2) of Proposition 3 claims that there is always a trivial market equilibrium. A trivial market equilibrium is not interesting, because if there is also a non-trivial market equilibrium then traders prefer a non-trivial market equilibrium allocation to a trivial one.

In the following, we focus on the possibility of non-trivial market equilibria and study their properties.

5. Properties of a market equilibrium under the state of mind $\boldsymbol{\lambda}$

5.1 Walrasian world : an extremely optimistic or optimistic state of mind — for all i and all h, $\lambda_h^i = \infty$ or $\lambda_h^i > 1$

In this section, we study the properties of a market equilibrium. We will see that in this mechanism depending on the distribution of values of the parameters λ, agents have a variety of trade intensity in a market equilibrium.

First, we show that the Walrasian world appears when all traders are either extremely optimistic or optimistic for all goods. In such a market state of mind, the (non-trivial) market equilibrium allocation is the one in the Walrasian mechanism. We define a market equilibrium in the Walrasian mechanism.

Definition 5.1

(a) Agent i's set of trades in the Walrasian mechanism is

$$\beta^{wi}(s^i) = \{z^i \in Z^i | p \cdot z^i = 0\}.$$

(b) Agent i's excess demand function in the Walrasian mechanism is

$$\zeta^{wi}(s^i) = \{z^i \in \beta^{wi}(p) \mid z^i \text{ maximizes } v^i(z^i)\}.$$

(c) A market equilibrium in the Walrasian mechanism is $(p^w, (\mathbf{z}^{wi*}))$ which satisfies the following two conditions:

Condition (i) for all $i = 1,\cdots,n$, $\mathbf{z}^{wi} = \zeta^{wi}(p^w)$,

Condition (ii) $z^w = \sum_i \mathbf{z}_h^{wi} \leqq 0$, $p^w \cdot z^w = 0$.

We consider the case in which every trader is extremely optimistic for all goods. In this case, the market equilibrium is clearly the one in the Walrasian mechanism.

Proposition 4

If all traders are extremely optimistic for all goods, that is, for all i and all h $\lambda_h^i = \infty$, then the (non-trivial) market equilibrium $(p^*, (\mathbf{z}^{i*}))$ under the state of mind λ is the market equilibrium $(p^w, (\mathbf{z}^{wi*}))$ in the Walrasian mechanism.

Proof

If all traders are extremely optimistic for all goods, agent i's trade opportunity set $\beta^i(s^i)$ coincides with the set of trades in the Walrasian mechanism $\beta^{wi}(p)$. Now, suppose that agent i's excess demand of good h is positive, that is, $\mathbf{z}_h^{i*} > 0$. The agent in question receives the quantity signal $z^*(i)$ and perceives that the half real line $[0, +\infty)$ is his trading opportunity for good h. Although a trader in the Walrasian mechanism perceives that his trading opportunity for good h is the whole real line $(-\infty, +\infty)$, by supposition he is interested in trading a nonnegative quantity of good h. Thus, utility maximization under the quantity constraint $\mathbf{z}_h^{i*}$ coincides with the Walrasian excess demand z_h^{wi}. This argument can be applied for all i and h.

Q.E.D.

As in the proof in Proposition 4, what makes this mechanism different from the Walrasian mechanism is just quantity constraints faced by agents. Other states of mind may realize the Walrasian allocation at a market equilibrium.

Proposition 5

If all agents are optimistic about trading all goods, that is, if for all i and all h, $1 < \lambda_h^{\mathrm{i}} < \infty$, then the (non-trivial) market equilibrium (p^*, $(\mathbf{z}^{i*})$) under the state of mind λ is the market equilibrium (p^{w}, $(\mathbf{z}^{\mathrm{w}i*})$) in the Walrasian mechanism.

Proof

As in the proof in Proposition 4, suppose that agent i's excess demand of good h is positive, that is, $\mathbf{z}_h^{i*} > 0$. The agent in question receives the quantity signal z^* (i). He perceives that his trading opportunity for good h is represented by the line segment $[0, -\lambda_h^{\mathrm{i}}\sum_{k \neq \mathrm{i}}\mathbf{z}_h^{k*}]$. Because the point $\mathbf{z}_h^{i*} = -\sum_{k \neq \mathrm{i}}\mathbf{z}_h^{k*}$ is an interior point of the line segment, the quantity constraint $-\lambda_h^{\mathrm{i}}\sum_{k \neq \mathrm{i}}\mathbf{z}_h^{k*}$ does not bind his trade. Thus, the excess demand $\mathbf{z}_h^{i*}$ under the state of mind λ is exactly the Walrasian excess demand $\mathrm{z}_h^{\mathrm{wi}}$. This is true for all agents and goods.

Q.E.D.

The argument above shows that a market equilibrium under the state of mind could realize the Walrasian allocation.

5.2 The world of a trivial market equilibrium: extremely pessimistic state of mind across the economy — for all i and h, $0 \leqq \lambda_h^{\mathrm{i}} < 1$

We have shown that the Walrasian world appears when everybody is either extremely optimistic or optimistic. Now, we show that the other extreme case exists, at least logically. As stated above, we are interested in studying the properties of a non-trivial equilibrium. However, there is a case in which there is not a non-trivial equilibrium.

Now, the following Proposition 6 claims that such a trivial market equilibrium appears in the case that every trader is extremely pessimistic. Statement (1) claims that if trader i is extremely pessimistic for good h, then he ends up with no trade of good h. Statement (2) claims that if all traders are extremely pessimistic for all goods, then a trivial market equilibrium appears.

Proposition 6

(1) If trader i is extremely pessimistic for good h at a market equilibrium (p^*, $(\mathbf{z}^{i*})$) under the state of mind λ, that is, if $0 \leqq \lambda_h^{\mathrm{i}} < 1$, then $\mathbf{z}_h^{i*} = 0$.

(2) If all traders are extremely pessimistic for all goods, that is if for all $\mathrm{i} = 1,\cdots,\mathrm{n}$ and $\mathrm{h} = 1,\cdots,\ell$, $0 \leqq \lambda_h^{\mathrm{i}} < 1$, then the market equilibrium (p^*, $(\mathbf{z}^{i*})$) under the state of mind λ is a trivial market equilibrium (p^*, (0)).

Proof

We show statement (1) by contradiction. Suppose that $\mathbf{z}_h^{i*} > 0$ at the market equilibrium (p^*, $(\mathbf{z}^{i*})$). Then $-\lambda_h^{\mathrm{i}}\sum_{k \neq \mathrm{i}}\mathbf{z}_h^{k*} < \mathbf{z}_h^{i*} = -\sum_{k \neq \mathrm{i}}\mathbf{z}_h^{k*}$. This implies that $\mathbf{z}^{i*} \notin \beta^{\mathrm{i}}(s^{i*})$. This is contrary to the assumption that $\mathbf{z}^{i*}$ is a market equilibrium allocation. The same argument holds when $\mathbf{z}_h^{i*} < 0$. Thus, $\mathbf{z}_h^{i*} = 0$.

Statement (2) follows by applying statement (1) for all traders and all goods.

Q.E.D.

The trivial market equilibrium case is not interesting, because everybody has no trade although everybody wants to trade. However, it is important in the sense that it shows the other extreme case against Walrasian world exists at least logically.

5.3 The world of quantity constrained market equilibria: pessimistic state of mind across the economy — for all i and h $\boldsymbol{\lambda}_h^{\mathbf{i}} = \mathbf{1}$

In this section, we consider the case every trader is pessimistic about trading all goods. In this case, the world of quantity-constrained equilibria appears, which has been studied under the assumption of quantity adjustment with fixed prices after Clower's "dual decision hypothesis." In this mechanism, perfect smooth price adjustment yields market equilibria in which the Walrasian excess demands are not zero.

Definition 5.2

(a) When at a market equilibrium (p^*, ($\mathbf{z}^{i*}$)) under the state of mind λ there is a trader i such that his excess demand for good h z_h^{i*} satisifies the following condition (*), trader i is "quantity constrained."
Condition (*) there exists z^i such that $z^i \in \beta^{Ni}$ (s^i $(-h)$) $= \{z^i \in Z^i | \, p \cdot z^i = 0, z_{k\_}^{i} \leqq z_k^i \leqq z_k^{i-}, k \neq h, \ell\}$ and $v^i(z^i) > v^i(\mathbf{z}^{i*})$.

(b) The budget set in the case that trader i is not quantity-constrained for good h is β^{Ni} (s^i $(-h)$) $= \{z^i \in Z^i | \, p \cdot z^i = 0, z_{k\_}^{i} \leqq z_k^i \leqq z_k^{i-}, k \neq h, \ell\}$. The excess demand function without quantity constraints for good h is $\zeta^{Ni}(s^i) = \{z^i \in \beta^{Ni}(s^i\ (-h)) \,|\, z^i$ maximizes $v^i(z^i)\}$. The excess demand without quantity constraints is $z^{Ni} = \zeta^{Ni}$ (s^i $(-h)$).

By Proposition 7, we show that when all traders are pessimistic about trading all goods market equilibria appear in which quantity constraints bind traders' trades effectively and such equilibria constitute a continuum.

Proposition 7

(1) If all traders are pessimistic for all goods, that is, if $\lambda_h^i = 1$ for all i and all h, and if there exists a market equilibrium (p^*, ($\mathbf{z}^{i*}$)) under the state of mind λ that is not trivial, then for any α, $0 \leqq \alpha \leqq 1$, (p^*, ($\alpha\mathbf{z}^{i*}$)) is also a market equilibrium under the state of mind λ.

(2) If all traders are pessimistic for all goods, that is, if $\lambda_h^i = 1$ for all i and all h, there exists trader i who is quantity-constrained on trading at least one good at a non-Walrasian market equilibrium (p^*, ($\mathbf{z}^{i*}$)) under the state of mind λ.

(3) If all traders are pessimistic for all goods, that is, if $\lambda_h^i = 1$ for all i and all h, and if there exists a market equilibrium (p^*, (z^{i*})) under the state of mind λ that is not trivial, there exists trader i who is quantity-constrained on trading at least one

good at a non-Walrasian market equilibrium (p*, $(\alpha \mathbf{z}^{i*})$) with $\alpha < 1$ under the state of mind λ.

(4) If there exists a market equilibrium (p*, $(\mathbf{z}^{i*})$) under the state of mind λ when all traders are pessimistic for all goods, that is, if $\lambda_h^{\mathrm{i}} = 1$ for all i and all h, then for some $\alpha \geqq 1$ there exists a trader who is not quantity-constrained for some good h in the market equilibrium (p*, $(\alpha \mathbf{z}^{i*})$).

Proof

Note that the existence theorem above ensures the existence of a market equilibrium under any state of mind λ.

First, we prove statement (1). Consider a non-trivial market equilibrium (p*, $(\mathbf{z}^{i*})$). Then by Statement (1) of Proposition 3 it holds that for all i and all h $z_h^{\mathrm{i}*} = -\sum_{k \neq \mathrm{i}} \mathbf{z}_h^{k*}$. If α is either 0 or 1, we know that (p*, $(\alpha \mathbf{z}^{i*})$) is also a market equilibrium under the state of mind λ; hence consider (p*, $(\alpha \mathbf{z}^{i*})$) for any α such that $0 < \alpha < 1$. From the fact that (p*, $(\mathbf{z}^{i*})$) is a market equilibrium, $z^* \leqq 0$, $p^* \cdot z^* = 0$. It follows that for any α, $\alpha z^* \leqq 0$, $\alpha p^* \cdot z^* = 0$. This is condition (ii) of a market equilibrium. It remains to show that condition (i) of a market equilibrium holds. Note that every trader receives the signal $s^{\mathrm{i}} = (p^*, (\alpha \mathbf{z}^* (\mathrm{i})))$. Because $\alpha z_h^{\mathrm{i}*} = \alpha(-\sum_{k \neq \mathrm{i}} \mathbf{z}_h^{k*})$, $\alpha z^{\mathrm{i}*} \in \beta^{\mathrm{i}}(s^{\mathrm{i}}) \subset \beta^{\mathrm{i}}(s^{i*})$. This inclusion holds because $s^{i*} = (p^*, (\mathbf{z}^* (\mathrm{i}))$. As $\mathbf{z}^{i*} = \zeta^{\mathrm{i}}(s^{i*})$, for any $\mathbf{z}^{i\prime} \in \beta^{\mathrm{i}}(s^{\mathrm{i}})$, $v^{\mathrm{i}}(\mathbf{z}^{i\prime}) < v^{\mathrm{i}}(\alpha \mathbf{z}^{i*}) < v^{\mathrm{i}}(\mathbf{z}^{i*})$. This holds for α close enough to 1. We can reiterate this process; hence (p*, $(\alpha \mathbf{z}^{i*})$) satisfies condition (i) of a market equilibrium. Therefore, (p*, $(\alpha \mathbf{z}^{i*})$) is a market equilibrium under the state of mind λ.

To prove statement (2), we note that there exist many non-Walrasian equilibria. By Statement (2) of Proposition 3, there is always a trivial market equilibrium. By applying Statement (1) of Proposition 7, the market equilibrium (p^{W}, $(\alpha \mathbf{z}^{\mathrm{Wi}})$) for any $0 < \alpha < 1$ is non-Walrasian. There are many non-Walrasian equilibria in this economy. Now, suppose that there is a non-Walrasian market equilibrium in question (p*, $(\mathbf{z}^{i*})$). Then, there exists at least one good of which Walrasian excess demand is not zero. Now suppose that $z_h^{\mathrm{w}}(p^*) > 0$. Then there exists i such that $\mathbf{z}_h^{\mathrm{wi}} > 0$. For this i, there exists z^{i}, for example $z^{\mathrm{wi}}(p^*)$ such that $z^{\mathrm{i}} \in \beta^{\mathrm{wi}}(p) = \{z^{\mathrm{i}} \in Z^{\mathrm{i}} | p \cdot z^{\mathrm{i}} = 0\}$ and $v^{\mathrm{i}}(z^{\mathrm{i}}) > v^{\mathrm{i}}(\mathbf{z}^{i*})$. The set of z^{i} with $v^{\mathrm{i}}(z^{\mathrm{i}}) > v^{\mathrm{i}}(\mathbf{z}^{i*})$ is open, so that there exists $\mathbf{z}^{\mathrm{i}\prime}$ that is sufficiently close to $z^{\mathrm{i}*}$, $\mathbf{z}^{\mathrm{i}\prime} \in \beta^{\mathrm{Ni}}(s^{\mathrm{i}}(-h)) = \{z^{\mathrm{i}} \in Z^{\mathrm{i}} | p \cdot z^{\mathrm{i}} = 0, z_{k\_}^{\mathrm{i}} \leqq z_k^{\mathrm{i}} \leqq z_k^{\mathrm{i}-}, k \neq h, \ell\}$, and $v^{\mathrm{i}}(\mathbf{z}^{\mathrm{i}\prime}) > v^{\mathrm{i}}(\mathbf{z}^{\mathrm{i}*})$. That is, trader i is quantity-constrained on trading good h.

In the case that Walrasian excess demand for good h is negative, that is $z_h^{\mathrm{w}}(p^*) < 0$, the same argument applies, and a quantity-constrained trader exists.

Statement (3) follows as an immediate consequence from Statements (1) and (2).

We prove statement (4) as follows. Condition (b) of Definition 5.2 defines the budget set $\beta^{\mathrm{i}}(s^{\mathrm{i}}(-h))$, the excess demand function $\zeta^{\mathrm{Ni}}(s^{\mathrm{i}}(-h))$, and the excess demand z^{Ni}, in which trader i is not quantity-constrained on trading good h. Suppose that there is a good h which all traders are quantity-constrained in trading. Defining α^* by $\alpha^* = \min[z_h^{\mathrm{N1}}/z_h^{1}, \cdots, z_h^{\mathrm{Ni}}/z_h^{\mathrm{i}}, \cdots, z_h^{\mathrm{Nn}}/z_h^{\mathrm{n}}]$, there exists a trader j such that $\alpha^* = z_h^{\mathrm{Nj}}/z_h^{\mathrm{j}}$. Because $z^{\mathrm{Nj}} = \alpha^* z^{\mathrm{j}}$, trader j is not quantity constrained. We still need to show that (p*, $(\alpha^* z^{\mathrm{i}*})$) is a market equilibrium, but it follows from the same argument as in the proof for statement (1).

Q.E.D.

5. 3. 1 Fixed-price quantity constrained market equilibrium

As mentioned above, the world in which all traders in the economy are pessimistic is similar to that which has been studied by fixed-price quantity constraint models. Contrary to appearances, these two mechanisms work very differently. In fixed-price quantity constraint models, traders are quantity-constrained because the prices are fixed. However, in a market economy under the state of mind λ, the price adjustment is disturbed because traders have the state of mind in which they may be quantity-constrained. Here, we compare these two mechanisms.

Definition 5.3

Given a price vector p, (p, ($\mathbf{z}^{i*}$)) is a "fixed-price wholly quantity-constrained market equilibrium" in which all traders have trade quantity functions $F_h^{\,i}(z_h(i)) = -\sum_{k \neq i} z_h^{\,k}$ for all goods and the following two conditions are satisfied:

(i) for every $i = 1,\cdots,n$, $\mathbf{z}^{i*} = \zeta^i(p, z^*(i))$,

(ii) $z^* = \sum_i z^{i*} \leqq 0$, $p \cdot z^* = 0$

Although it is apparent that there exists a fixed-price wholly quantity-constrained market equilibrium, we show it by means of Proposition 8.

Proposition 8

Assume that for all traders the following assumptions are satisfied:

Assumption (1') Z^i is closed, convex, and bounded from below.

Assumption (2') utility function $v^i:Z^i \rightarrow R$ is continuous, strongly quasi-concave, and has no bliss point.

Assumption (3') $0 \in \mathrm{int}Z^i$.

If all traders are pessimistic for all goods, then for any price vector p, there exists an allocation (z^{i*}) such that (p, ($\mathbf{z}^{i*}$)) is a market equilibrium under the state of mind λ. That is, the market equilibrium (p, ($\mathbf{z}^{i*}$)) is a fixed price wholly quantity-constrained market equilibrium.

Proof

To answer the question of whether there exists a market equilibrium in a quantity adjustment mechanism for a fixed-price economy, we must prove its existence theorem. However, the proof of this proposition is formally similar to that of the existence theorem in Section 3.

Given a price vector p, we consider the excess demand function $\tilde{\zeta}$ (p, z) : $\tilde{Z} \rightarrow \tilde{Z}$ which is a continuous function from a non-empty compact, convex set to itself. Thus, there exists a fixed point $z^* = \tilde{\zeta}(p, z^*)$. ($p^*$, ($\mathbf{z}^{i*}$)) is a market equilibrium under the state of mind λ.

Q.E.D.

Strictly speaking, because a fixed-price economy and a variable-price economy work completely differently, it is very difficult to compare their market equilibria. In the Walrasian mechanism, the initial price vector does not matter. A market equilibrium is determined only by its initial endowment. However, when all traders have pessimistic states of mind, the given price vector with the initial distribution of quantity signals sent by traders leads to a market equilibrium. With a different situation, we have a different equilibrium. In a fixed-price economy, the prices remain the same and only quantity adjustment takes place. Compared with this, in a variable-price economy in which all traders are pessimistic, not only quantity adjustment but also price adjustment works, so that the initial price vector does not necessarily remain the same. Despite this, we have the following equivalence theorem that compares market equilibria in these different mechanisms.

Proposition 9

Assume that all traders are pessimistic.

(1) If there exists a market equilibrium $(p^*, (\mathbf{z}^{i*}))$ under the state of mind λ, the allocation $(\mathbf{z}^{i*})$ can be realized as a fixed-price wholly quantity-constrained market equilibrium under the price vector p^*.

(2) A fixed-price wholly quantity-constrained market equilibrium $(p, (\mathbf{z}^{i*}))$ under the price vector p can be realized as a market equilibrium under the state of mind λ.

Proof

In both cases, we need to show that a given type of market equilibrium satisfies conditions of the other type of market equilibrium.

To prove the statement (1), we need to show that a market equilibrium under the state of mind λ satisfies conditions (i) and (ii) in Definition 5.3.1. By Proposition 3, the allocation $(\mathbf{z}^{i*})$ satisfies the condition $z_h^{i*} = -\sum_{k \neq i} z_h^k$ for all i and for all h. Because every trader i maximizes his utility under these quantity constraints, condition (i) of Definition 5.3.1, that is, $\mathbf{z}^{i*} = \zeta^i\ (p^*, z^*\ (i))$ for all i, holds. Condition (ii) of a market equilibrium of Definition 4, that is, $z^* = \sum_i z_h^{i*} \leqq 0$, $p^* \cdot z^* = 0$, is exactly condition (ii) of Definition 5.3.1 with the price vector p^*.

We can prove statement (2) similarly. By Statement (1) of Proposition 3, $z_h^* = 0$ for all h so that the price vector remains at p. Thus, $(p, (\mathbf{z}^{i*}))$ is a market equilibrium under the state of mind λ.

Q.E.D.

Proposition 9 claims that the set of market equilibria under the state of mind λ when all traders are pessimistic is equivalent to the set of fixed-price wholly quantity-constrained equilibria. However, we must reiterate that these two market mechanisms work differently.

5. 3. 2 The world dominated by the "short-side rule"

A fixed-price quantity-constrained model describes the world dominated by the "short-side rule." In the framework of a partial equilibrium of one good, the demand curve determines the quantity of trade when the price is above the equilibrium price, and the supply curve determines the quantity of trade when the price is under the equilibrium price. To describe such a world, we need to define the sides of a market.

Definition 5.4

The sum of excess demands without quantity constraints of good h is denoted by $z_h^{N} = \sum_i z_h^{Ni}$.

(a) If $z_h^{N} \cdot z_h^{Ni} > 0$, then trader i is on the "long-side" of market h.

(b) If $z_h^{N} \cdot z_h^{Ni} < 0$, then trader i is on the "short-side" of market h.

If the sign of trader i's excess demand without quantity constraints of good h is the same as that of market excess demands without quantity constraints, then he is on the long-side of the market. If the signs are opposite, then trader i is on the short-side of the market. A fixed-price quantity-constrained model claims that a trader on the short-side of the market is free from quantity constraints and can trade as much as his utility-maximizing excess demand. However, a trader on the long-side of the market is quantity-constrained.

If we assume that quantity signals are the major determinants of the state of mind, then we show that a market equilibrium under the state of mind λ can be considered as a state in the world dominated by the short-side rule.

Assumption (4) (quantity signals and the state of mind)

Assume that the following state of mind dominates all commodity markets, $h = 1, \cdots, \ell - 1$:

(a) If trader i is on the long-side of market h, then he is pessimistic, that is $\lambda_h^{i} = 1$.

(b) If trader i is on the short-side of market h, then he is optimistic, that is $\lambda_h^{i} > 1$.

With this assumption, we obtain the following proposition.

Proposition 10

Assume Assumption (4). If in a market equilibrium $(p^*, (\mathbf{z}^{i*}))$ under the state of mind λ trader i is on the short-side of all commodity markets, then $\mathbf{z}_h^{i*} = z_h^{Ni}$ for $h = 1, \cdots, \ell - 1$.

Proof

We show this by contradiction. Suppose that there is trader i who is on the short-side of market h and $0 < \mathbf{z}_h^{i*} < z_h^{Ni}$. Then by Statement (1) of Proposition 3 we have $z_h^{i*} = -\sum_{k \neq i} z_h^{k*}$. By Assumption (4), the trader is optimistic so that $z_h^{i*} = -\sum_{k \neq i} z_h^{k*} < -\lambda_h^{i} \sum_{k \neq i} z_h^{k*} \leqq z_h^{Ni}$. Then $\mathbf{z}_h^{i\prime} = -\lambda_h^{i} \sum_{k \neq i} z_h^{k*} \in \beta^{Ni}(s^i) = \{z^i \in Z^i \mid p \cdot z^i = 0, z_{h\_}^{i} \leqq z_h^{i} \leqq z_h^{i-}, h \neq \ell\}$ and $\mathbf{z}_h^{i*} = \zeta_h^{i}(s^{i*})$. It follows that $v^i(\mathbf{z}_h^{i*}) > v^i(\mathbf{z}_h^{i\prime})$. However, because $z_h^{Ni} \in \zeta^{Ni}(s^i)$, $v^i(\mathbf{z}_h^{i*}) < v^i(z_h^{Ni})$ and $\mathbf{z}_h^{i\prime} \in [\mathbf{z}_h^{i*}, z_h^{Ni}]$. This contradicts the quasi-concavity of a utility function. Thus, there is no trader who is on the short-side of the market when $0 < z_h^{i*} < z_h^{Ni}$ holds. Similarly, we can show that there is no trader on the short-side when $0 > z_h^{i*} > z_h^{Ni}$ holds. Hence, in any commodity market short-side $\mathbf{z}_h^{i*} = z_h^{Ni}$ holds for any trader on the short-side of the market.

Q.E.D.

Proposition 10 claims that every trader on the short-side of a market is not quantity-constrained if the exchange of quantity signals leads traders to the state of mind so as to satisfy Assumption (4). Many traders on the long-side of a market are quantity- constrained. Assumption (4) presumes that every trader learns correctly the whole market situation through quantity signals and he is under his state of mind based on what he has learned. For traders to have such rational expectations, there must be two processes: one from quantity signals to the state of a market, and one from the state of a market to the state of mind. In the world dominated by the short-side rule, the workings of these two processes should be included in the mechanism. To show that such a mechanism works, we need a model with these two processes and need to show the existence of a market equilibrium. In this paper, we presume that the state of mind is formed based on past experiences and is independent of the current exchange of quantity signals. Indeed this is one difference between the market mechanism of fixed-price quantity-constrained models and the mechanism under consideration in this paper. There is a much larger difference between these two mechanisms, which we consider in the next section. We will see that the world never appears in a fixed-price quantity-constrained model.

5. 4 Gray world: an amalgam of optimistic, pessimistic, and extremely pessimistic traders

Up to this point, we have presumed a drastic situation in which all traders are under identical states of mind for all goods although each trader may have different trade experiences in the past. It is important to learn how differently our mechanism could perform. Indeed we know that in one case our mechanism behaves in exactly the same way as the Walrasian mechanism and in the other case it behaves like a fixed-price quantity- constrained economy. However, we also know that the real world always looks gray. As seen above, each trader must have a different trade experience; hence it is not strange that traders on the same side of a market may be under different states of mind. In particular, after an extraordinary incident or at the time when so many things are happening, traders' states of mind must be a mixture of optimism, pessimism, and extreme pessimism even if they may be in the same kind of state of mind at the end. In this section, we examine what will happen if traders' states of mind are mixed.

Proposition 11

(1) If in a market equilibrium $(p^*, (\mathbf{z}^{i*}))$ under the state of mind λ trader i is extremely pessimistic for good h, that is, $0 \leqq \lambda_h^i < 1$, then $z_h^{i*} = 0$.

(2) If in a market equilibrium $(p^*, (\mathbf{z}^{i*}))$ under the state of mind λ trader i is pessimistic for good h and the Walrasian aggregate excess demand for good h at the price vector p^* is not zero, that is, $z_h^w(p^*) \neq 0$, then there exists at least one trader j who is quantity-constrained in the market equilibrium.

Proof

Statement (1) is just the restatement of statement (1) of Proposition 6.

To show statement (2), we note that there is at least one trader j such that $z_h^{j*}(p^*) \neq \mathbf{z}_h^{wj}(p^*)$ due to the assumption $z_h^w(p^*) \neq z_h^*(p^*) = 0$. For this trader j, there exists

z^j, which is sufficiently close to z^{j*}, $z^j \in \beta^{Nj}(s^j(-h)) = \{z^j \in Z^j | p \cdot z^j = 0, z_{k\,-}^{\,j} \leqq z_k^{\,j} \leqq z_k^{\,j-}$, $k \neq h, \ell\}$ and $v^j(z^j) > v^j(z^{j*})$. Thus, trader j is quantity-constrained in trading good h.
Q.E.D.

Statement (1) of Proposition 11 is the restatement of statement (1) of Proposition 6. Statement (2) of Proposition 11 is the restricted version of statement (2) of Proposition 7 to one commodity market. Statements (1), (3) and (4) of Proposition 7 cannot be restricted to one commodity market. Keeping the trades of other goods, if the amount of trade of one commodity is changed, then a budget constraint is not satisfied and the resulting allocation with a given price vector cannot be a market equilibrium.

Through Proposition 11, we consider the gray world in which traders are under a variety of states of mind and the market economy is divided by separate markets. We need the following notations. The set of whole traders is $N = \{1,\cdots,n\}$. N^O is the set of extremely optimistic and optimistic traders in all markets. N^{EP} is the set of traders extremely pessimistic in trading all goods, and the set of traders pessimistic in all markets is N^P. Then, $N = N^O \cup N^{EP} \cup N^P$. Here, we note that any product set of these subsets is empty.

We consider a market economy that consists of sub-economies dominated by different states of mind. We denote economies by the set of traders as follows: E (N), E (N^O), E (N^{EP}), E (N^P). The market equilibrium under the state of mind λ for these economies should be denoted by [(p^*, z^*) ; E (N)], [(p^*, z^*) ; E (N^O)], [(p^*, z^*) ; E (N^{EP})], [(p^*, z^*) ; E (N^P)]. By Propositions 4 and 5, a market equilibrium under the state of mind λ for E (N^O) is exactly the same as a Walrasian equilibrium. That is, we denote [(p^*, z^*) ; E (N^O)] by [(p^W, z^W) ; E (N^O)]. Similarly we denote equilibrium price vectors by [p^*; E (N)], [p^*; E (N^O)], [p^*; E (N^{EP})], [p^*; E (N^P)], [p^W; E (N^O)], and equilibrium allocations by [z^*; E (N)], [z^*; E (N^O)], [z^*; E (N^{EP})], [z^*; E (N^P)], [z^W; E (N^O)]. Using these notations, we describe a divided market economy as follows.

Proposition 12

In an economy E ($N = N^O \cup N^{EP}$), a market equilibrium [(p^*, z^*) ; E (N)] under the state of mind λ consists of the Walrasian price vector [p^W; E (N^O)] and the combination of allocations, that is, the Walrasian allocation [z^W; E (N^O)] in E (N^O) and the trivial allocation [z^*; E (N^{EP})] = [(0) ; E (N^{EP})] in E (N^{EP}). That is, [(p^*, z^*) ; E (N)] = ([p^W; E (N^O)], {[z^W; E (N^O)], [(0) ; E (N^{EP})]}).

Proof

We show statement (1). By Propositions 4 and 5, it is apparent that there exists a market equilibrium under the state of mind λ in an economy E (N^O), and it coincides with Walrasian equilibrium. By statement (2) of Proposition 6, a market equilibrium [(p^*, z^*) ; E (N^{EP})] in an economy E (N^{EP}) is the trivial market equilibrium. Note that this claim is valid regardless of a market equilibrium price vector. Thus, there exist market equilibria [(p^W, z^W) ; E (N^O)] and ([p^W; E (N^O)], [z^*; E (N^{EP})]) = ([p^W; E (N^O)], (0)) in economies E (N^O) and E (N^{EP}) and the equilibrium price vector is in common. It remains to show that the combination of ([p^W; E (N^O)], {[z^W; E (N^O)],

$[(0)\,;E\,(N^{EP})]\})$ satisfies the equilibrium conditions in the whole economy E $(N = N^{O} \cup N^{EP})$.

Condition (ii) of a market equilibrium under the state of mind λ requires that $z^* = \sum_{i \in N} z^{i*} \leqq 0$ and $p^* \cdot z^* = 0$ hold in the economy E (N). In the economy E (N^{O}), $[z^*; E\,(N^{O})] = \sum_{i \in N^O} z^{iW} \leqq 0$, $p^W \cdot z^W = p^W \cdot [z^*; E\,(N^{O})] \leqq 0$, and in the economy E (N^{EP}) $[z^*; E\,(N^{EP})] = \sum_{i \in N^{EP}} z^{i*} = 0$, $p^W \cdot z^* = p^W \cdot [z^*; E\,(N^{EP})] \leqq 0$. Thus, it is clear that $z^* = \sum_{i \in N} z^{i*} = \sum_{i \in N^O} z^{iW} + \sum_{i \in N^{EP}} z^{i*} \leqq 0$ and $p^* \cdot z^* = p^* \cdot \sum_{i \in N^O} z^{iW} + p^* \cdot \sum_{i \in N^{EP}} z^{i*} = 0$.

To show that condition (i) of a market equilibrium under the state of mind λ is satisfied by $([p^W; E\,(N^{O})], \{[z^W; E\,(N^{O})], [(0)\,; E\,(N^{EP})]\})$, it is enough to show that quantity constraints in the partial economies E (N^{O}) and E (N^{EP}) remain valid in the whole economy E (N). In the economy E (N^{O}), no trader is quantity-constrained, and in the economy E (N^{EP}), every trader has just a trivial allocation regardless of quantity signals. Thus, the allocation consists of $[z^*; E\,(N^{O})]$, and $[z^*; E\,(N^{EP})]$ is a market equilibrium allocation in the economy E (N).

Proposition 12 claims that in an economy which consists of two sub-economies, E (N^{O}) and E (N^{EP}), we have just one market equilibrium, in which agents in E (N^{O}) trade actively in the market and agents in E (N^{EP}) exit from the market. They are completely divided.

Corollary to Proposition 12

In an economy E $(N = N^{O} \cup N^{EP} \cup N^{P})$, $[(p^*, z^*)\,; E\,(N)] = ([p^W; E\,(N^{O})], \{[z^W; E\,(N^{O})], [(0)\,; E\,(N^{EP})], [z^*; E\,(N^{P})]\})$ is a market equilibrium $[(p^*, z^*)\,; E\,(N)]$ under the state of mind λ, where $[(p^*, z^*)\,; E\,(N)] = ([p^W; E\,(N^{O})], \{[z^W; E\,(N^{O})], [(0)\,; E\,(N^{EP})], [z^*; E\,(N^{P})]\})$ consists of the Walrasian price vector $[p^W; E\,(N^{O})]$ and the combination of allocations, that is, a fixed-price quantity-constrained market equilibrium allocation $[z^*; E\,(N^{P})]$ under the price vector $[p^W; E\,(N^{O})]$ in addition to the Walrasian allocation $[z^W; E\,(N^{O})]$ in E (N^{O}) and the trivial allocation $[z^*; E\,(N^{EP})] = [(0)\,; E\,(N^{EP})]$ in E (N^{EP}).

Proof

As seen in the proof of Proposition 12, $([p^W; E\,(N^{O})], [z^W; E\,(N^{O})])$ and $([p^W; E\,(N^{O})], [(0)\,; E\,(N^{EP})])$ are market equilibria under the state of mind λ in the economies E (N^{O}) and E (N^{EP}), respectively. We note that in the economy E (N^{P}), there exist a variety of and a continuum of market equilibrium allocations, including a trivial one. Thus, $([p^W; E\,(N^{O})], \{[z^W; E\,(N^{O})], [(0)\,; E\,(N^{EP})], [(0)\,; E\,(N^{P})]\})$ is a market equilibrium $[(p^*, z^*)\,; E\,(N)]$ under the state of mind λ. Furthermore, we have a fixed-price quantity constraint market equilibrium allocation $[z^*; E\,(N^{P})]$ under the price vector $[p^W; E\,(N^{O})]$. By statement (2) in Proposition 9, any $([p^W; E\,(N^{O})], [z^*; E\,(N^{P})])$ is a market equilibrium under the state of mind λ in the economy E (N^{P}), where $\sum_{i \in N^P} z^{i*} = 0$ holds.

It remains to show that $([p^W; E\,(N^{O})], \{[z^W; E\,(N^{O})], [(0)\,; E\,(N^{EP})], [z^*; E\,(N^{P})]\})$ satisfies the conditions of a market equilibrium under the state of mind λ in the economy E (N).

As for condition (ii) of a market equilibrium, in the sub-economy $E(N^O)$ we have $[z^*; E(N^O)] = \sum_{i \in N^O} z^{iW} \leqq 0$ and $p^W \cdot z^W = p^W \cdot [z^*; E(N^O)] \leqq 0$, in the sub-economy $E(N^{EP})$ we have $[z^*; E(N^{EP})] = \sum_{i \in N^{EP}} z^{i*} = 0$ and $p^W \cdot z^* = p^W \cdot [z^*; E(N^{EP})] \leqq 0$, and in the sub-economy $E(N^P)$ we have $[z^*; E(N^P)] = \sum_{i \in N^P} z^{i*} = 0$, $p^W \cdot z^* = p^W \cdot [z^*; E(N^P)] \leqq 0$. It follows that $[z^*; E(N)] = [z^*; E(N^O)] + [z^*; E(N^{EP})] + [z^*; E(N^P)] \leqq 0$ and $p^W \cdot [z^*; E(N)] = p^W \cdot [z^*; E(N^O)] + p^W \cdot [z^*; E(N^{EP})] + p^W \cdot [z^*; E(N^P)] = 0$.

To show that $([p^W; E(N^O)], \{[z^W; E(N^O)], [(0); E(N^{EP})], [z^*; E(N^P)]\})$ satisfies condition (i) of a market equilibrium, it is enough to show that quantity constraints in the partial economies $E(N^O)$, $E(N^P)$ and $E(N^{EP})$ remain valid in the whole economy $E(N)$. However, by statement (1) of Proposition 3, $[z^*; E(N^O)] = 0$, $[z^*; E(N^{EP})] = 0$, and $[z^*; E(N^P)] = 0$ in their sub-economies. Hence, the quantity constraints determined by equilibrium quantity signals in these sub-economies remain the same in the whole economy $E(N)$. Thus, the trading opportunity each trader faces is the same and he maximizes his utility subject to his trading opportunity. The allocation $([z^*; E(N^O)], [z^*; E(N^{EP})], [z^*; E(N^P)])$ is utility-maximizing allocation in the whole economy $E(N)$. This concludes the proof of condition (i). Thus, $([p^W; E(N^O)], \{[z^W; E(N^O)], [(0); E(N^{EP})], [z^*; E(N^P)]\})$ is a market equilibrium $[(p^*, z^*); E(N)]$ under the state of mind λ.

Q.E.D.

Corollary to Proposition 12 describes the world as follows. The market economy could be divided into sub-economies of these groups of traders: optimistic traders, pessimistic traders, and extremely pessimistic traders. Only in the sub-economy $E(N^O)$ of optimistic traders, the prices are fully flexible and the resulting allocation $[z^W; E(N^O)]$ in the sub-economy $E(N^O)$ is optimal among optimistic traders. Because the prices $[p^W; E(N^O)]$ are in common, pessimistic traders take the prices as given, trade subject to quantity-constrained trading opportunity, and have the allocation $[z^*; E(N^P)]$. Furthermore, traders in the sub-economy $E(N^{EP})$ lose incentive to trade, trade nothing, and has the allocation $[(0); E(N^{EP})]$. In the divided market economy shown in Proposition 12 and its corollary, the flexible prices do not make it possible for a whole economy to utilize available resources and improve the economic welfare.

The world shown in Proposition 12 or its corollary is indeed an extreme one. A more probable world is as follows. A trader is optimistic and an almost-perfect competitor in trading most goods. However, he has experienced irretrievable failures in the past and cannot be positive in trading some goods. It is easy to find examples of such irrevocable failures in our daily life. Expecting a large demand wave, a retail merchant may have had a huge inventory of some consumer goods. Inexperienced workers or workers with outdated skills may have failed to find employment repeatedly, lost the incentive to continue job hunting, and exited from the labor market. In a financial market, many inexperienced investors invest large amounts of money in risky assets at the time of a bubble or a big boom, end up having huge debts, and then decide not to invest anymore. At the time of huge changes, a market economy produces failures like these so many that they are widespread and known universally.

Now, we may call such a world a “weakly gray world” where every trader is pessimistic or extremely pessimistic in trading some goods, but there are some optimistic traders for all goods. In such an economy, there is a potential malfunction in the price adjustment mechanism. A trader, who is pessimistic or extremely pessimistic and are quantity- constrained in some goods, may be even more positive in trading the other goods. From the viewpoint of optimal utilization of resources, the market mechanism fails to play the role in proportion as the color of the world turns gray or black.

6. Policy implications

Assuming that the Walrasian mechanism works, we start from the fundamental theorem of welfare economics when we consider economic policies. Under certain combinations of economic environments and economic systems, a perfect competitive market economy realizes a Pareto optimal resource allocation. Consequently, the theorem claims that if we live in a market economy with such a combination of an economic environment, an economic system, and the market structure, the market economy assures us of our best economic welfare. This implies that if any conditions concerning an economic environment, an economic system, and the market structure are different from the required conditions, then market failures occur. And we need some policies. In this paper, we claim that the state of mind can be another factor of market failures.

As seen in Section 5, only if every trader is optimistic or extremely optimistic for all goods, our mechanism will work as if it is the Walrasian mechanism. If any trader is pessimistic or extremely pessimistic in trading any goods, then the price adjustment mechanism deteriorates into malfunctions. In a situation described in Proposition 11 or 12, the prices are not responsive to either the whole traders in some markets or some group of traders in the whole economy, so that the price adjustment mechanism develops a malfunction, and there is room to improve with some policies. At the end of Section 5, we mentioned “potential malfunctions” in a gray world, which is like a lifestyle illness. By correcting such malfunctions, economic welfare improves.

Then what kind of policies will correct malfunctions of the price adjustment mechanism that we have considered in this paper? There are at least three kinds of policies. The most important policy is “primary education,” which helps anyone develop the ability to continue being a sound trader even during a period of huge changes. We must know that we face various risks such as a wave of technical innovations, a change in population structure, investment fluctuation of houses and production equipment, demand waves by a fashion, advertisement, and political changes including wars. We ought to realize that we need some preparation for these risks. Without such preparation, a trader may fail and obtain a critical damage. Education of history in a broad sense, in particular education of economic history, is the core of primary education here. A new technology creates a sudden boom. A boom may become a bubble. Any boom ends, and any bubble collapses. This simple fact is well-known. However, people still go wild at the time of a boom or a bubble and some always experience huge damage. In this sense, we have to keep learning from such incidents as long as we live.

The second kind of policy is “technical education,” which assures traders of not failing in trading[14]. In consumer good markets, it is highly probable that there is a huge difference in the quality and quantity of information of goods, and in bargaining power between consumers and business. To avoid buying a good of poor quality or being the victim of a fraud, a consumer must examine the quality of goods and use a cooling-off period, for example. To protect consumers from bad contracts and unethical business practice, consumer education may help consumers develop self-reliance. In a labor market, a worker should know that each step in his career development requires a different skill. Without a knowledge of what skills are needed when, a worker is unprepared in the labor market. Career education should help a worker in being self-reliant socially and professionally and in having the necessary fundamentals to develop his career[15]. In a financial market, an investor should learn the characteristics of financial commodities and how to take steps of risk dispersion through financial and investor education[16]. During periods in which the market environments are stable, parents could teach their children simple aspects of market trading. However, today, things change so drastically and rapidly not only in a financial market and a labor market but also in a consumer goods market. In any market, failure in market trading is inevitable without the technical education mentioned here. The above mentioned primary education and technical education reduce the chances of traders' failures, which make them pessimistic in market trading.

The third kind of policy is “encouragement.” Anyone who has failed needs encouragement to stand up again. For obvious malfunctions as shown in Propositions 11 and 12, he may be helped by government intervention, including government assistance by public funds. If malfunctions affect the whole market economy, we need a Keynesian countercy-

14 We take up consumer education, career education and investor education as examples of technical education mentioned here. As for consumer education, through the period of rapid economic growth in Japan (late 1950-1960s), the necessity of consumer education has been recognized. Providing consumer education and promoting public relations for consumer protection became a basic consumer protection measure. In 1968, the Basic Consumer Protection Act was enacted. However, a number of government agencies and departments worked for consumer education on a section-by-section basis. Since the beginning of the 21st century, a “safe and secure market” or a “market with quality” had been called for. The Basic Consumer Protection Act was renamed the Basic Consumer Act and was enacted in 2004. Furthermore, a number of consumer agencies and departments in administration were integrated into the Consumer Agency in 2009. Now, consumer education has been provided as a part of school education. Various campaigns for enlightenment have been conducted, particularly in May, which is the so-called the “consumer month.” Since 1990, the National Institute on Consumer Education has been working on promoting consumer education as a specialty. However, the White Paper on the National Lifestyle 2008 evaluated that efforts so far have not achieved that much. Other technical education options such as career education and investor education are also a part of school education. Various governmental agencies and departments are working toward enlightenment activities.

15 The Association for Technical and Career Education defines the terms of vocational (or job) education and career education as follows: Vocational education is one that cultivates the knowledge, skills, abilities, and attitude required to follow a certain or specific job. Career education is one that aims to let each person be self-reliant socially and professionally, cultivates the required fundamentals of abilities and attitude, and promotes career development.

16 Japan Securities Dealers Association has been playing a major role to promote investor education in Japan.

clical measure, which is government intervention including monetary and fiscal policies. It should be taken for a long enough period to change traders' states of mind, but not for long enough to cause moral hazard. Failures in price adjustment may be the largest market failure; hence we should not hesitate to implement any government intervention to alleviate the situation.

This last kind of policy reflects Samuelson's neoclassical synthesis. That is, the price adjustment mechanism does not work so well in the short run that recession may take place. If we get rid of the GDP gap using monetary and fiscal policies and traders' states of mind improve, then the price adjustment mechanism would recover from malfunctions. In the long run, the price adjustment mechanism ensures a market economy on the path of economic growth.

7. Conclusion

This paper has examined the price adjustment mechanism, that is the working of a market economy under perfect competition. One of requirements of the concept "perfect competition" is an agent to be a price-taker. First, an agent is an "atomic existence," which is small relative to the whole economy; hence he is not capable of manipulating the prices. Second, an agent has a horizontal subjective demand curve and a limitless trading opportunity at the going prices. In the beginning of this paper, we stressed that these two requirements are different and independent.

A limitless trading opportunity is a belief that an agent can trade limitless amount or more than his capability of trading. Such a belief should be supported by firm confidence in market trading. We consider the temporary equilibrium framework in this paper to examine consequences of the fact that there are occasions when a market economy faces a huge change. Such radical changes in a market economy inevitably cause successes and failures to traders. A successor gains huge profits and firm confidence. A loser from the same change must meet with gigantic losses and discouragement. A dis-hearted trader cannot be optimistic enough to believe that his trading opportunity should be limitless. A pessimistic trader must think that he can trade no more than the amount offered now. An extremely pessimistic trader may believe that he can trade less than the current offer.

In a market economy in which an atomic existence has a variety of states of mind, either the Walrasian or a quantity-constrained mechanism works. That is, passive trading by a pessimistic trader leads the price adjustment mechanism embodied by an auctioneer to a malfunction, because extremely pessimistic traders will exit from market trading, and pessimistic traders see the remaining demand as his trading opportunity. As the numbers of these traders increases, the market trading shrinks, that is, the quantity signals become smaller in the absolute value. The price signals based on these quantity signals cannot be an invisible hand, which is supposed to lead a whole market economy to no waste use of economic resources. This is the market failure in this paper.

We need some kinds of policy to remedy a market failure. The market failure examined in this paper is caused by a heavy failure in market trading and resulting discouragement. Consequently, we propose three steps of policy. First, we should provide traders by primary education to be prepared in a market economy. Second, we should train traders to be successful in market trading. And finally, government intervention should support discouraged traders. Malfunctions of the price adjustment mechanism are the most fundamental market failure; hence a variety of measures should be taken.

We must stress the market failure discussed in this paper is to provide a microfoundation of macroeconomics. Today, the mainstream economists discuss microfoundations of macroeconomics in the framework of the Walrasian mechanism. However, as we point out, a Walrasian market economy is a special case in which there are only optimistic traders who have firm confidence in market trading. In a changeful world, there are always some traders who fail in market trading and get dis-hearted. Some countercyclical measures by fiscal and monetary policies are necessary at least temporarily. By realizing a temporary boom, pessimistic traders will recover confidence in market trading. That will remedy the malfunction of the price adjustment mechanism. If the price adjustment mechanism is restored, then the market economy will be on its path of economic growth.

Finally, we point out the subjects we left. Although we have shown that pessimistic states of mind should cause a malfunction of the price adjustment, we did not describe how the malfunction occurs. In the case in which a trader is either pessimistic or extremely pessimistic in trading some goods, there must be some spill-over effect that is resulting demand and supply of the other goods caused by unfilled demand and supply of the good in question. The spill-over effect distorts price adjustment in the Walrasian mechanism. It is very important to know how this distortion takes place. The study on this point will be an assignment for the future.

Another issue we left in this paper is a stability analysis. In this paper, we suppose that a trading opportunity is limited by upper and lower bounds as in the fixed-price quantity constraint models. There are necessarily kinks, that are in the way of a full stability analysis in the framework of a differential equation system. It is very important to know whether a market equilibrium is stable if not only price adjustment but also quantity adjustment works. To discuss the point, we need to construct a model in which there is no kink point in a trading opportunity function. This will be another assignment for the future.

The last assignment we left is to draw macroeconomic implications of this model. Because our model is stimulated by the studies of microeconomic foundations of macroeconomics, it is interesting to know how the market mechanism in this paper works in a macroeconomic framework. We have shown that there is a situation, in which the price adjustment mechanism works in some sectors of an economy and in the other sectors the price adjustment mechanism does not work but the quantity adjustment mechanism works. A macroeconomic model of such economy is new and interesting, because new classical macroeconomic models consider the price adjustment only and in fixed-price quantity-constrained models the price adjustment halts for some reasons. However, it is difficult to consider the distribution of states of mind in a macroeconomic framework, in which there are a representative consumer, a representative producer, and a government. We must find a proxy variable of the distribution of states of mind and explore macroeconomic implications. Again, this is an assignment for the future.

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